

Questions

(32 points in total)

1. (5 points) Let $A = \begin{bmatrix} a & 3 \\ 2 & -5 \end{bmatrix}$ be a matrix.

a) If A is singular, then what is the value of a ?

b) Calculate A^{-1} for $a = 0$.

2. (10 points) Let $A = \begin{bmatrix} 2 & -1 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$ be a matrix over rationals.

(a) Find A^{-1} for $a \neq 0$. Write out A^{-1} as a product of elementary row matrices.

(b) Write out the equation $A\vec{x} = \vec{b}$ as a system of linear equations.

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 19 \\ 4 \\ 12 \\ c \end{bmatrix}$$

(c) Solve $A\vec{x} = \vec{b}$ for (i) $a = 1, c = 1$, (ii) $a = 1, c = 0$, (iii) $a = 0, c = 1$.

3. (7 points) Find a least squares solution to the system:

$$\begin{cases} x + y = 7 \\ x + z = 14 \\ y + z = 21 \\ x + y + z = 28 \end{cases}$$

Write out the steps to the solution. You can use external tools to find matrix products and inverses.

4. (10 points) Find a matrix that corresponds to a projection on

a) a line that goes through the origin at an angle α to the x -axis in two-dimensional space.

b) a plane defined by points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, \sqrt{3})$.